The Mathematics of Big Data

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Big data

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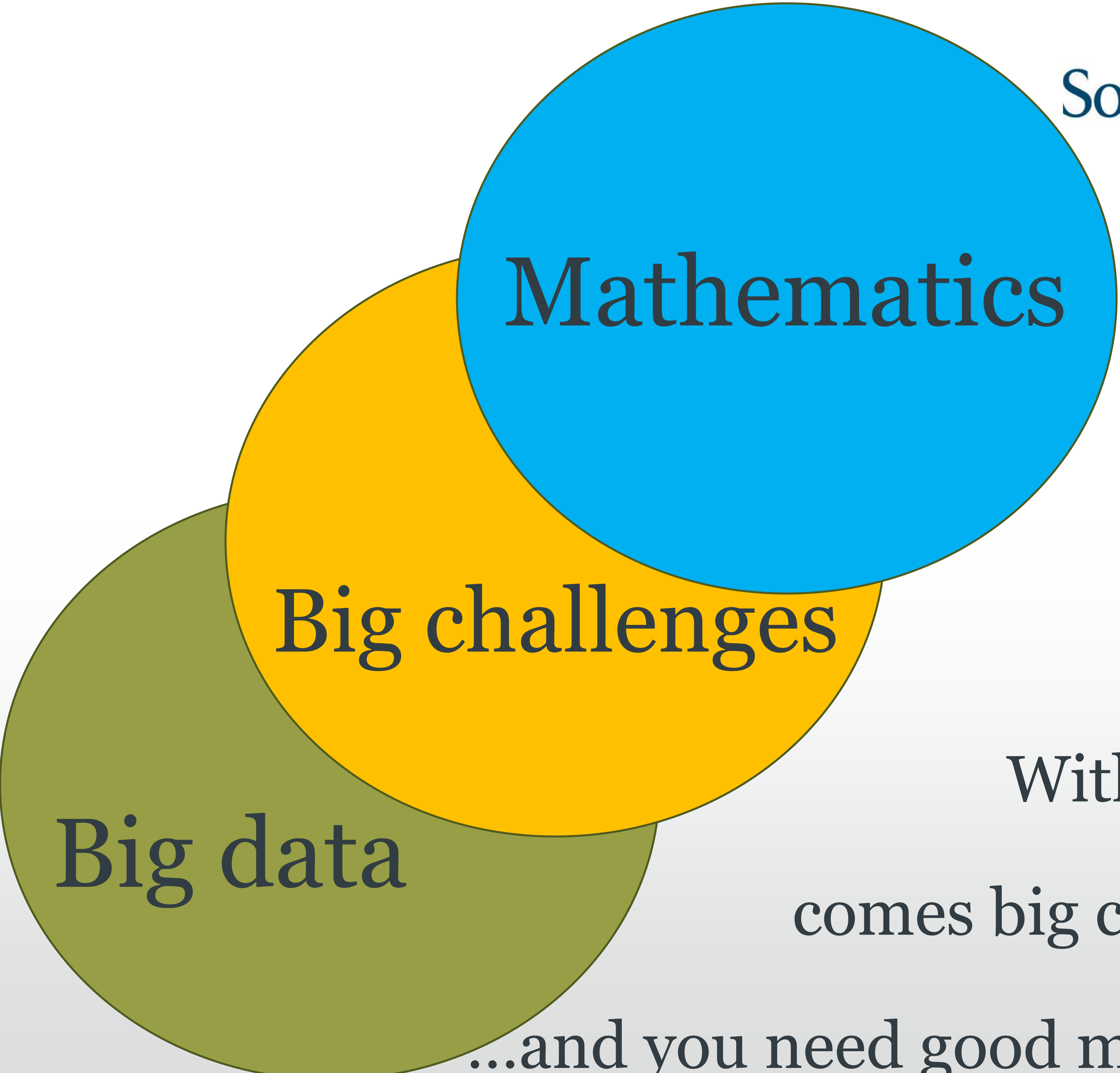

With big data...

comes big challenges...

...and you need good mathematics

Big data: challenges

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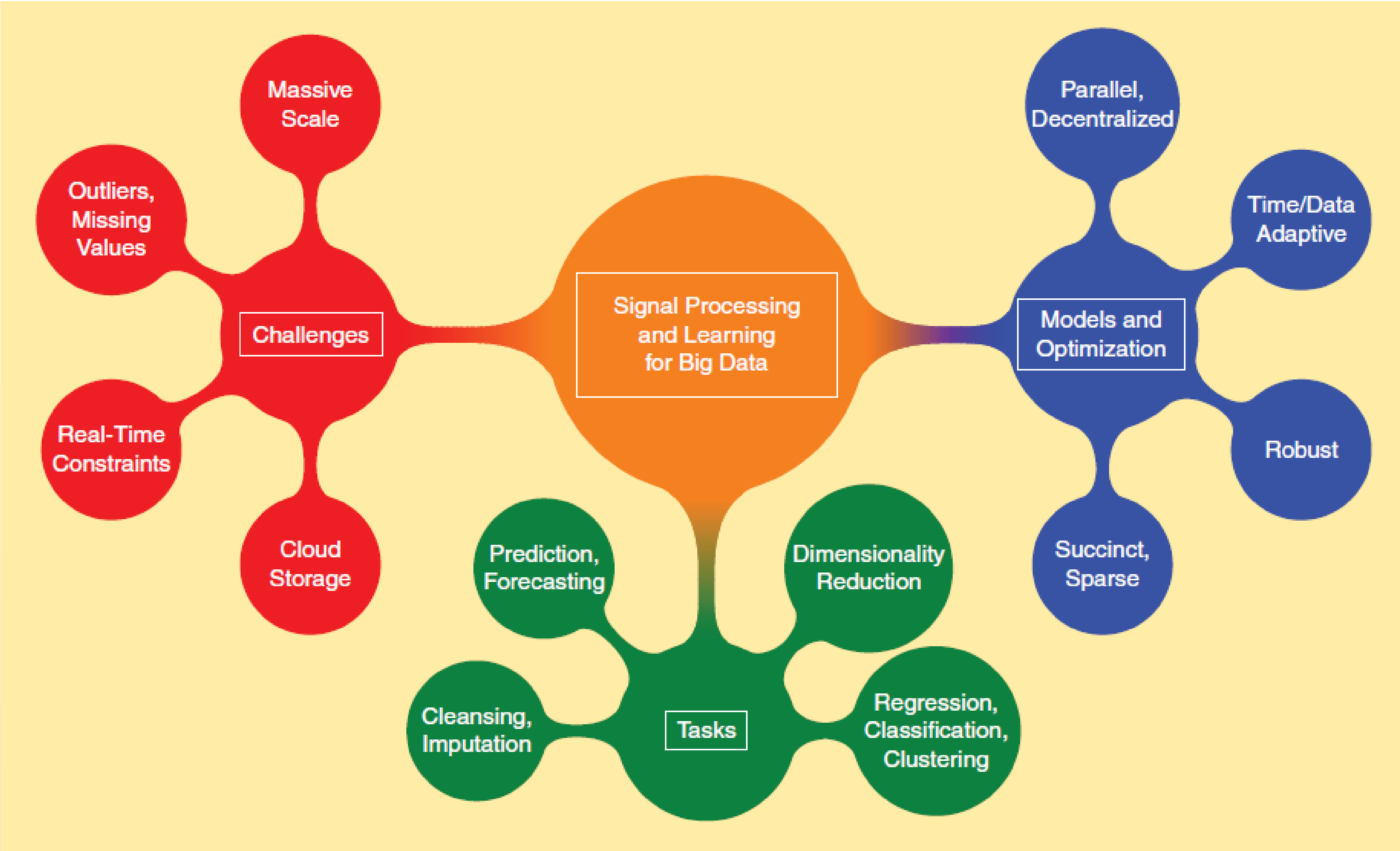
- Central processing infeasible
- Central storage infeasible
- Streaming data: real-time learning
- Streaming: no revisiting of past entries
- Need to revisit old tools from signal processing and statistical learning

Big data: challenges, tasks, and optimization



Example

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(Ex.: traffic data, N traffic links, T time slots.)

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& modelling/measurement errors $V \in \mathbb{R}^{N \times T}$

Solve

$$Y \approx L + DS + V$$

with some "dictionary matrix" D.

But not all entries of Y are important, so use a projection operator and solve

$$\mathcal{P}(Y) \approx \mathcal{P}(L + DS + V)$$

But how do we model L low rank and S sparse?

Write the task as an optimisation problem:

$$\min_{L,S} \|\mathcal{P}(Y - L - DS)\|_F + \lambda\|L\|_* + \omega\|S\|_0$$

Weight λ controls rank penalty.

Weight ω controls sparsity penalty.

Consider

$$\min_{L,S} \|\mathcal{P}(Y - L - DS)\|_F + \lambda \|L\|_* + \omega \|S\|_0$$

Weight λ controls rank penalty.

Weight ω controls sparsity penalty.

One rich, versatile model that explains data parsimoniously and succinctly.

$$\min_{L,S} \|\mathcal{P}(Y - L - DS)\|_F + \lambda\|L\|_* + \omega\|S\|_0$$

This approach subsumes

- Principle component analysis, robust PCA
- Dictionary learning
- Compressed sampling, compressed sensing
- Subspace clustering
- Nonnegative matrix factorization
- Missing value imputation
- Regression
- Kernel-based learning
- Dimensionality reduction

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One mathematical model to rule ~~them all~~
a lot of other approaches

Algorithms

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$$\min_{L,S} \|\mathcal{P}(Y - L - DS)\|_F + \lambda\|L\|_* + \omega\|S\|_0$$

- ADMM: alternating direction method of multipliers
- DR: Douglas-Rachford algorithm
- BCDM: block-coordinate descent methods
- K-SVD
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Decentralized

Parallelizable

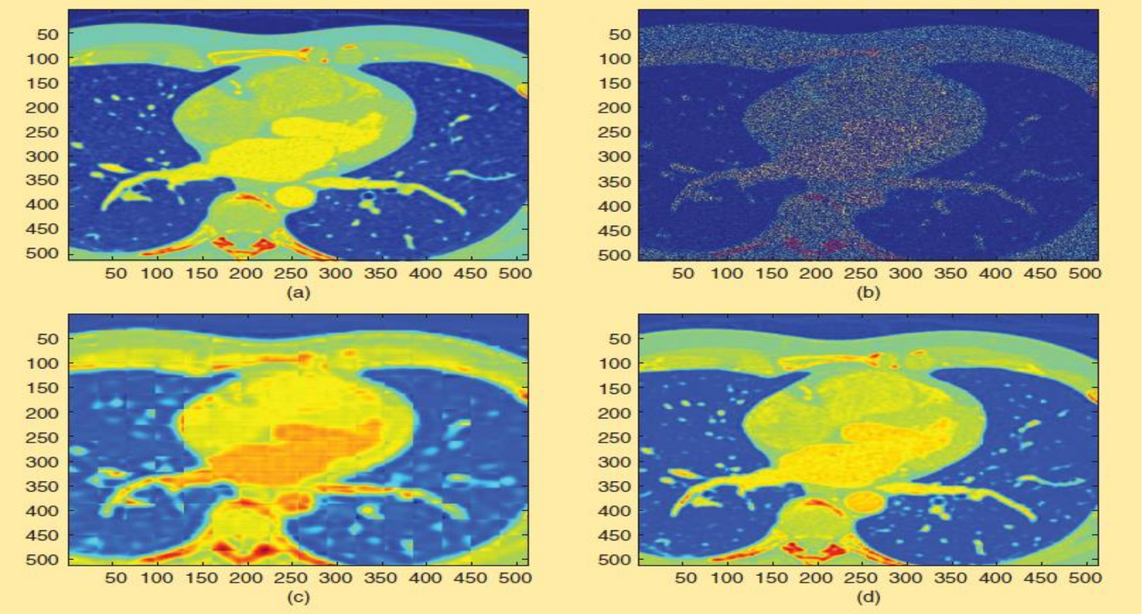
Robust

Online

Scalable

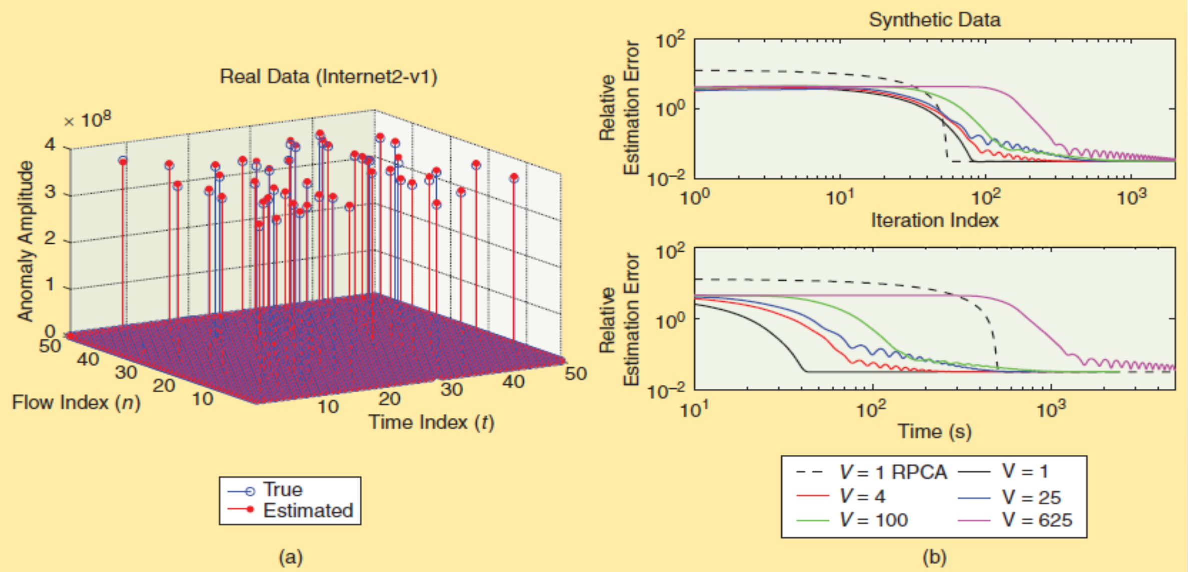
Convergence guarantee: we know they always work!

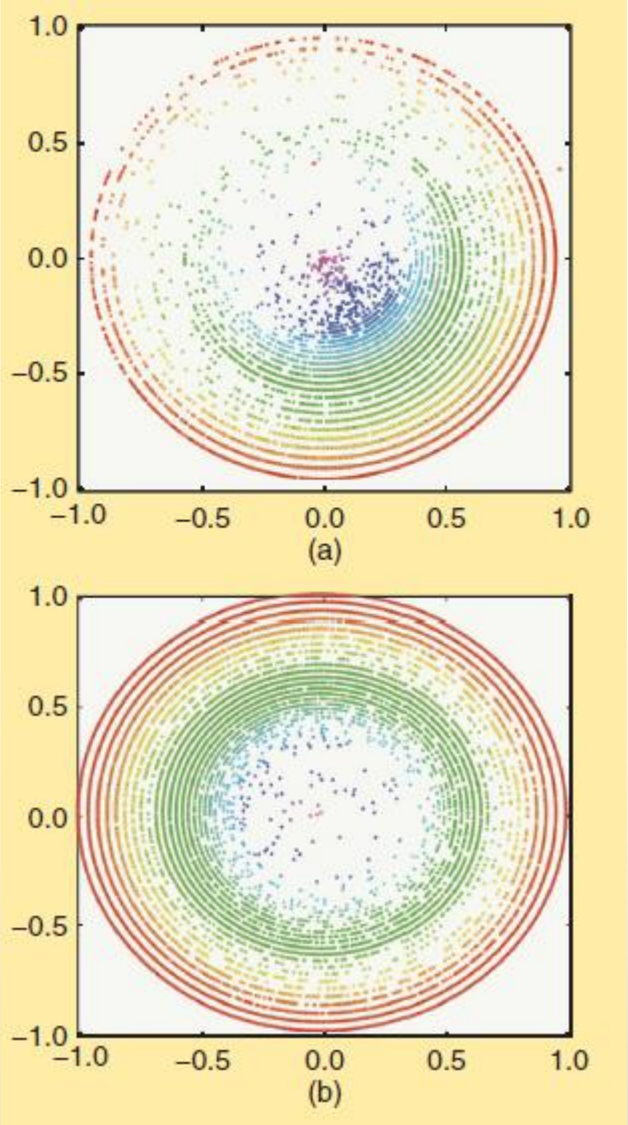
Applications



Imputation of MRI data

Traffic: outlier detection & analysis





Dynamic network visualization

Conclusions

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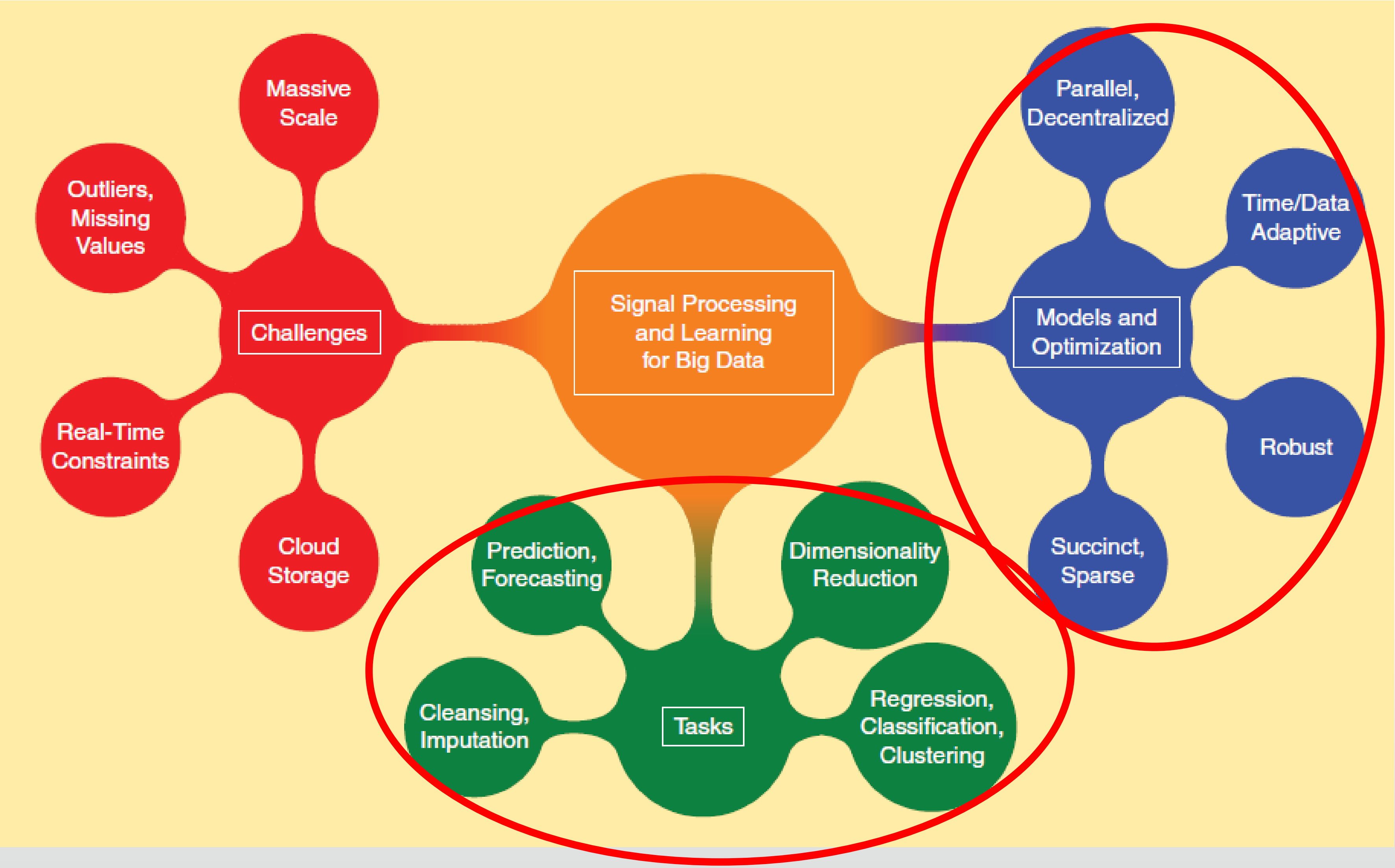

Mathematics: we are here to help.